Multiparameter Optimization of Inverse Filtering Algorithms

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Abstract—This paper investigates inverse filtering of transient signals. The problem is ill-conditioned, which means that a small uncertainty in the measurement causes large deviations in the reconstructed signal. This amplified noise has to be suppressed at the price of bias in the estimation.

The most difficult task is to find the optimal degree of noise reduction. Deconvolution algorithms are usually controlled by one or a few parameters. Several algorithms can be found in the literature to find the best setting of inverse filtering methods; however, usually methods with only one free parameter are handled.

In this paper, an algorithm is proposed to optimize several parameters, on the basis of a spectral model. Multiparameter inverse filtering methods have the advantage that they can be better adapted to the measurement system, and to the noise and signal to be measured. The superiority of the proposed optimization method is demonstrated both on simulated and on experimental data.

I. INTRODUCTION

MEASUREMENT of transient signals with large bandwidth requires special effort. If the bandwidth of the measurement system is not high enough, the observed waveform will be a distorted version of the original one. In such a case the measured signal should be postprocessed to get accurate results, by compensating the effect of the measurement system. This procedure is called inverse filtering. When the measurement system can be modeled as a linear and time-shift invariant one, the relation between the input and the output signals can be described by convolution. In this special case inverse filtering is called deconvolution. In this paper the latter case will be investigated.

The model of the measurement system is depicted in Fig. 1. Unfortunately, the inverse filtering problem is ill-posed, which means that small uncertainties in the output signal, caused by the noise, lead to great differences in the estimated input. The amplified noise has to be suppressed at the price of bias in the estimate. Several algorithms are proposed to compensate the effect of the measurement system, with simultaneous noise
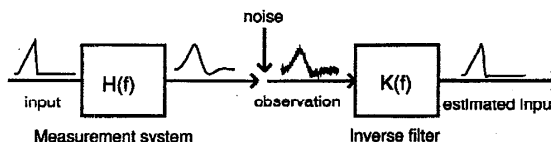
Fig. 1. Model of the measurement and the inverse filtering system.

suppression ([1]–[10]). They are mostly controlled by one or two parameters [e.g., cutoff frequency of a lowpass filter, regularization parameter(s) etc.].

The most difficult problem is to select the optimal setting of the parameters to find the tradeoff between the bias and the variance in the estimate. The existing techniques usually handle one parameter only, which restricts the shape of the inverse filter. We propose a model-based optimization technique where the number of parameters is only limited by the quality of the spectral signal models. It makes possible to compensate the effect of the measurement system with more complicated inverse filters than the single-parameter methods. The superiority of our algorithm will be demonstrated on an extended regularization method, for both simulated and experimental data.

II. PREVIOUS OPTIMIZATION TECHNIQUES

In the first attempts to do optimal inverse filtering, simple methods were used (e.g., lowpass filtering after compensating the effect of the measurement system), and the tradeoff parameter was adjusted manually, according to the a priori knowledge about the signal shape, smoothness etc.

Nahman proposed a heuristic method to find the best input estimation, using frequency-domain regularization [4]. The optimal setting is that for which the standard deviation of the imaginary part of the estimated signal after the inverse Fourier transform is minimal. The convergence depends on the implementation of the discrete Fourier transform.

Parruck *et al.* calculated the optimal deconvolution for impulse response data [5]. A steplike waveform is derived by integrating the impulse response. The tail part of the calculated step response is investigated. The optimum setting is that for which the empirical mean value and the empirical standard deviation of the tail part of the step response satisfy certain conditions. Although the optimum satisfies the conditions, manual interaction is required, since the solution is not unique.

Bertocco *et al.* investigated steplike waveforms [6], based on the a priori knowledge that the output reconstruction noise should have a uniform distribution in the whole record length. They separated the signal into two parts, a front one and a tail one. The optimum is that for which the average squared output reconstruction errors in the two parts are equal. The solution is unique; however, the optimization is proposed for only one parameter regularization method.

Younan *et al.* developed an optimization method for a lowpass filtering method [7]. They investigated the residual of the filtered observation by a randomness test. This method gives satisfactory results; however, the lowpass nature of the measurement system and the signal, and the wide-band nature of the measurement noise are assumed.

Dhaene *et al.* have presented a technique to optimize two parameters [8]. They tried to find the best set of parameters for the regularization method, where both energy and smoothness constraints are incorporated into the error criterion. The frequency range is separated into pass bands, attenuation bands and stop bands. They defined noise factors which should be calculated for a grid of parameters in every band. The noise factors should satisfy certain conditions, which can be evaluated on three-dimensional graphs or contour plots. Unfortunately, manual interaction is required to separate the spectra into bands and to evaluate the three-dimensional graphs.

III. MODEL-BASED MULTIPARAMETER OPTIMIZATION TECHNIQUE

We propose an optimization method to find the best set of parameters for inverse filtering methods. The optimum is defined as the input estimation which has the least squared error. The method is based on modeling of the magnitude of the signal spectra. Using an approximate model for the spectra of the input signal and of the noise, the squared error of the estimated input signal (error energy) can be approximately calculated in the frequency domain. The best parameter set is defined for which the approximate error energy is minimal.

In the following it will be assumed that the data are available in sampled form. The error energy of the estimated input signal can be calculated both in the time and frequency domain using Parseval's theorem

$$EE = T_s \sum_{n=0}^{N-1} [x(n) - x_{est}(n)]^2$$
$$= \frac{T_s}{N} \sum_{n=0}^{N-1} |X(k) - X_{est}(k)|^2 \qquad (1)$$

where EE is the error energy, $x(n)$ is the input signal, $x_{est}(n)$ is the estimated input, T_s is the sampling period, N is the number of sampled points, and the capital letters correspond to the DFT's of the signal sequences. Here the problem is that $x(n)$ is unknown, so for the minimization of (1) we have to make some approximations. Assuming an additive output noise source and substituting the explicit form of the spectrum

of the estimated signal into (1) we get

$$\begin{aligned} EE &= \frac{T_s}{N} \sum_{k=0}^{N-1} |X(k) - [X(k)H(k)K(k) \\ &\quad + N(k)K(k)]|^2 \\ &= \frac{T_s}{N} \sum_{k=0}^{N-1} |X(k)[1 - H(k)K(k)]|^2 \\ &\quad + \frac{T_s}{N} \sum_{k=0}^{N-1} |N(k)K(k)|^2 \\ &\quad - \frac{2T_s}{N} \sum_{k=0}^{N-1} |X(k)[1 - H(k)K(k)]| \\ &\quad \cdot |N(k)K(k)| \cos[\varphi(k)] \end{aligned} \qquad (2)$$

where $H(k)$ is the transfer function of the measurement system, $K(k)$ is the transfer function of the inverse filter, $N(k)$ is the DFT of the noise sequence, and $\varphi(k)$ is the phase angle of the two terms in the last sum. The error energy is thus split into three terms

$$EE = EE_{bias} + EE_{noise} + EE_{bias,noise} \qquad (3)$$

where the bias term is due to the distortion of the useful signal, the noise term is due to the variance, and the $EE_{bias,noise}$ term is due to the cross relation of the previous two terms. We approximate the error expression by two steps:

1) the $EE_{bias,noise}$ term is neglected, and
2) instead of the absolute values of the signal and noise spectra an approximate spectral model will be substituted into $X(k)$ and $N(k)$.

In [9] it was shown for deconvolution methods with one free parameter that under certain conditions the shape of the error function is not significantly modified with the above approximations; thus the location of the minimum of the error function remains nearly the same. The approximate error energy is calculated for a set of parameter combinations, and the location of the minimum is taken as the best parameter setting of the inverse filter. The input signal is then reconstructed using the optimal parameters. For the model building, an iterative algorithm was proposed in [9]. The main idea of the algorithm is that the magnitude of the DFT of the observation is a satisfactory model for the input signal if the distortion is not extremely high. However, the spectral model can be refined, if the model of the input signal is substituted with the magnitude of the DFT of the previous input estimation. After a few iterations (5–10), the fluctuation of the parameter is small enough, and the last estimation is accepted as the optimum. The noise can usually be modeled well by a white spectrum (constant magnitude).

In this paper, it will be shown that the method is suitable to find the best input estimation even if more than one parameter should be optimized. The advantage of a deconvolution method with more free parameters is that it can be better

adapted to the measurement system, input signal and noise. However, care should be taken to select the number of free parameters, because only rough information is given about the spectra of the useful signal and of the noise. Too many free parameters would cause the incorporation of the spectral modeling errors into the input estimation.

This optimization algorithm can generally be used for every kind of deconvolution method. We will demonstrate its abilities in the regularization method.

A. Regularization of the Transfer Function

The ill-posedness of the inverse filtering problem can be well illustrated in the frequency domain. If we compensate the effect of the measurement system, the noise will be amplified at those frequencies where the measurement system has large suppression

$$X_{est}(f) = \frac{Z(f)}{H(f)} = X(f) + \frac{N(f)}{H(f)} \qquad (4)$$

where $X_{est}(f)$ corresponds to the estimated input signal, and $Z(f)$ corresponds to the noisy observation. Equation (4) is the solution if the prediction error is minimized in the least squares sense. However, if the error criterion is redefined, a well-posed problem can be obtained by minimizing not only the prediction error but its sum with another regularization function. If this function is the energy of the input signal, the following inverse filter can be obtained:

$$K(f) = \frac{H(f)^*}{|H(f)|^2 + \lambda} \qquad (5)$$

where λ is the regularization parameter which controls the noise reduction, and superscript * denotes complex conjugation. The parameter λ puts a limit to the noise amplification, because the denominator becomes lower bounded.

Instead of the energy constraint, any derivative of even order can be incorporated into the error criterion. Moreover, they can be taken into account simultaneously. We define an inverse filter for which the energy, the smoothness, and the fourth-order derivative of the input signal are incorporated into the error criterion

$$Cost = ||z(i) - y_{pred}(i)|| + \lambda ||x_{est}(i)||$$
$$+ \gamma ||L^2 x_{est}(i)|| + \delta ||L^4 x_{est}(i)|| \qquad (6)$$

where $y_{pred}(i)$ denotes the predicted output signal, calculated from the estimated input, $|| \cdots ||$ denotes the l_2 norm, L^2 denotes the second-order, and L^4 denotes the fourth-order backward difference operator. The parameters λ, γ, and δ control the ratio of the different terms. The above criterion leads to the inverse filter

$$K(f) = \frac{H(f)^*}{|H(f)|^2 + \delta |L(f)|^4 + \gamma |L(f)|^2 + \lambda} \qquad (7)$$

where $L(f)$ is the Fourier transform of the second-order backward difference sequence. According to our experience, this inverse filter can adapt to a very wide range of signals and measurement systems. The results are generally better than those obtained from the regularization method with one parameter.
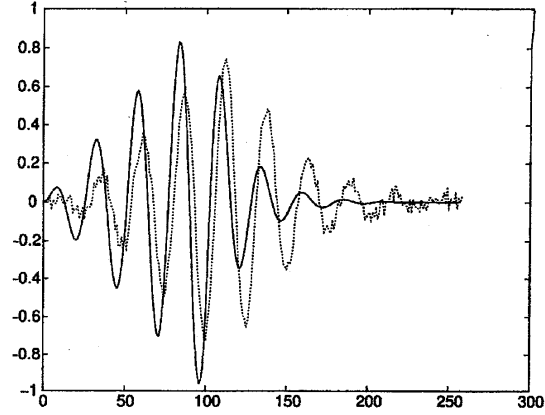

Fig. 2. Bandpass measurement system with a narrow-band input. Input signal (solid line), noisy output signal (dotted line).
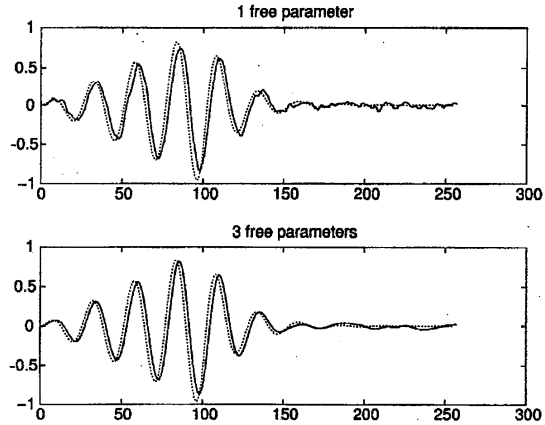

Fig. 3. Reconstructed signals, true input (dotted line), estimated input (solid line, shifted right with 2 points). Upper figure: one-parameter regularization (bounded energy). Lower figure: three-parameter regularization.

IV. SIMULATION RESULTS

In order to illustrate the effect of the three-parameter regularization method, a bandpass measurement system with a narrow-band input signal will be investigated. Normally distributed white noise is added to the output of the system (Fig. 2). The output SNR (relating the energies in the given time interval) is 20 dB.

Using the multiparameter optimization technique we calculated the error energy of the estimated input signal. The minimum of the three-dimensional error surface was found by common numerical optimization techniques (e.g., gradient, Newton–Gauss or Levenberg–Marquardt method [10]). The optimal estimation was calculated for both the three- and one-parameter regularization method (bounded energy). Using the one-parameter regularization method (bounded energy) an acceptable result is obtained (Fig. 3, upper figure), but the tail part contains small oscillations, caused by the noise. With the three-parameter method a smooth estimation was obtained, similar to the true input (lower figure). The squared error was decreased to one third.
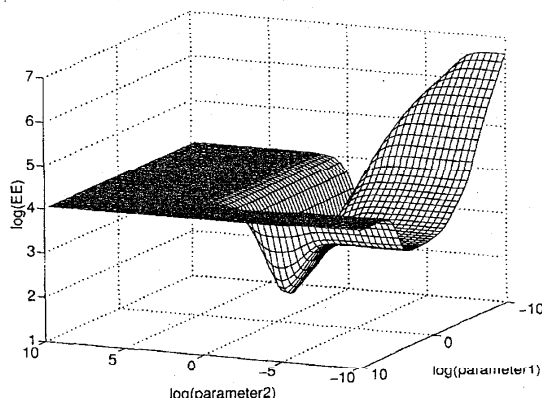
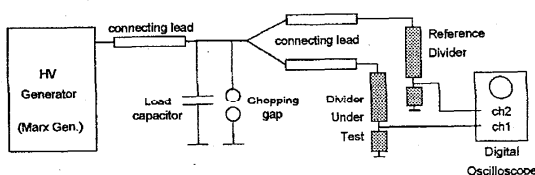Fig. 4. Error surface of the two-parameter regularization method.

Fig. 5. Diagram of the high-voltage measurement setup.

The error surface can be visualized for two parameters. Optimizing only one parameter means that the location of the minimum is seeking on the right edge of Fig. 4 (parameter2 is set to zero), which differs from the global minimum. (The same is valid, if parameter2 is minimized, and parameter1 is set to zero.) The global minimum can be found by optimizing more parameters together.

V. EXPERIMENTAL VERIFICATION

Testing insulators with lightning impulses is a common method in high-voltage engineering [11]. A large voltage of several hundred kilovolts up to 4 MV with duration of 2–200 μs is applied to the insulator, e.g., power transformers. From the shape of the measured signal the insulation can be verified and a possible fault source can be determined, by comparing the waveforms measured at full- and reduced-voltage levels. Such large voltages can only be measured through a special high-voltage divider. The common dividers often fail to satisfy the demands of the large bandwidth. In these cases digital postprocessing of the measured data can improve the accuracy.

We investigated a damped capacitive divider, which is suitable for measuring signals with a very wide range of duration. However, since a general-purpose capacitive divider has limited bandwidth, the divider fails to properly measure front-chopped lightning impulses. We compensated the effect of the divider by the three-parameter regularization method, and compared the results to the signals, measured with a low-ohmic reference divider. The measurements were made at a reduced level, about 60 kV peak value. The measurement setup is shown in Fig 5.
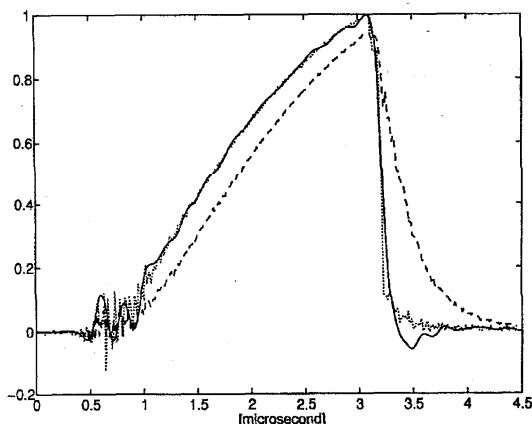

Fig. 6. Measured and reconstructed HV lightning impulse. Time from start to chopping is about 2.5 μs. Reference divider (dotted line), divider under test (dashed line), reconstruction (solid line).
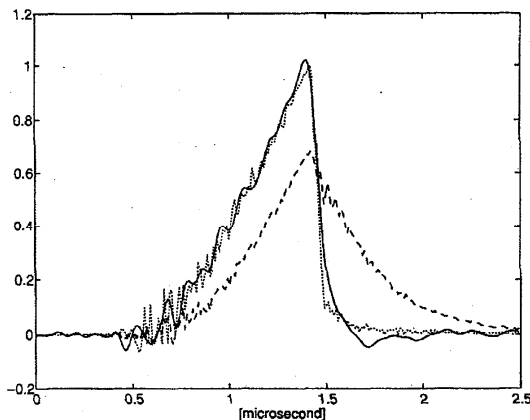

Fig. 7. Measured and reconstructed HV lightning impulse. Time from start to chopping is about 0.7 μs. Reference divider (dotted line), divider under test (dashed line), reconstruction (solid line).

The measured and reconstructed signals are depicted in Fig. 6. The estimated signal is very close to the output of the reference divider. The optimal parameter set is the following: $\delta = 385$, $\gamma = 2.3$, $\lambda = 0.0027$.

If the duration of the signal is shortened, the optimal parameter set is the following: $\delta = 1.9$, $\gamma = 0.51$, $\lambda = 0.0013$. The improvement is now more significant (Fig. 7). This can be seen well from the error of the important parameters of the signals. The peak value was measured with the investigated divider with 32% error, while it was decreased to 2.1%. Similarly, the rising slope was determined from the measurement of the divider under test with an error of 17%, which was improved to 1.4%.

VI. CONCLUSIONS

Inverse filtering of transient signals was investigated. We proposed a model-based optimization algorithm to select the

best parameter set of inverse filtering algorithms. The number of parameters to be optimized is only limited by the available information about the signals. The superiority of deconvolution methods with several free parameters was demonstrated. The performance of the multiparameter optimization algorithm was verified on both simulated and experimental data.

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REFERENCES

[1] A. N. Tikhonov and V. Y. Arsenin, *Solution of Ill-Posed Problems.* New York: Wiley, 1977.
[2] P. B. Crilly, "A quantitative evaluation of various iterative deconvolution algorithms," *IEEE Trans. Instrum. Meas.,* vol. 40, no. 3, pp. 558–562, 1991.
[3] B. D. O. Anderson and J. B. More, *Optimal Filtering.* Englewood Cliffs, NJ: Prentice Hall, 1979.
[4] N. S. Nahman, "Software correction of measured pulse data," in *Fast Electrical and Optical Measurements,* NATO ASI Series, T. E. Thompson and L. H. Luessen, Eds. Dordrecht, The Netherlands: Martin Nijhoff, 1986, pp. 351–417.
[5] B. Parruck and S. M. Riad, "An optimization criterion for iterative deconvolution," *IEEE Trans. Instrum. Meas.,* vol. IM-32, no. 10, pp. 137–140, 1983.
[6] M. Bertocco, C. Narduzzi, C. Offelli, and D. Petri, "An improved method for iterative identification of bandlimited linear systems," in *IEEE Instrumentation and Measurement Technol. Conf.,* Atlanta, GA, May 14–16, 1991, CH2940-5/91, pp. 368–372.
[7] N. H. Younan, A. B. Kopp, D. B. Miller, and C. D. Taylor, "On correcting HV impulse measurements by means of adaptive filtering and deconvolution," *IEEE Trans. Power Delivery,* vol. 6, no. 2, pp. 501–506, 1990.
[8] T. Dhaene, L. Martens, and D. De Zuttert, "Generalized iterative frequency domain deconvolution technique," in *IEEE Instrumentation and Measurement Technol. Conf.,* Irvin, Orange Country, CA, May 18–20, 1993, 93CH3291-0, pp. 85–87.
[9] T. Dabóczi, "Deconvolution of transient signals," Ph.D. dissertation, Technical University of Budapest, available as Tech. Rep. of the Technical University of Budapest, Ser. Elect. Eng., no. TUB-TR-94-EE12.
[10] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes, The Art of Scientific Computing.* Cambridge: Cambridge University Press, 1988.
[11] R. Malewski and B. Poulin, "Impulse testing of power transformers using the transfer function method," *IEEE Trans. Power Delivery,* vol. 3, no. 2, pp. 476–483, 1988.

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